MATH2230 Complex Variables with Application
Suggested Solution for HW10

SEC. 94 Hint:

6. (a) Let $f(z) = -5z^4$, $g(z) = z^6 + z^3 - 2z$. Then use Rouché's Thm.

   (b) Let $f(z) = 9$, $g(z) = 2z^4 - 2z^3 + 2z^2 - 2z$. . . . .

   (c) Let $f(z) = -4z^3$, $g(z) = z^7 + z - 1$. . . . .

7. Hint: (a) Let $f(z) = 9z^2$, $g(z) = z^4 - 2z^3 + z - 1$.
   Then use Rouché's Thm.

   (b) Let $f(z) = z^5$, $g(z) = 3z^3 + z^2 + 1$.
   Then use Rouché's Thm.

8. Solution: First, consider the number of zeros, counting multiplicities in the circle $C_1 : |z| = 1$.

   Let $f_1(z) = -6z^2$, $g_1(z) = 2z^5 + z + 1$.

   Noted that $f_1(z)$ and $g_1(z)$ are analytic on $|z| \le 1$ and that

   $$6 = |f_1(z)| > 4 = 2 \cdot 1^5 + 1 + 1 \ge |g_1(z)| \quad \text{on } |z| = 1.$$

   By Rouché's Thm., we have that $2z^5 + z - 6z^2 + 1$ has 2 zeros, counting multiplicities inside C_1.

   Next, we conside zeros, counting multiplicities in the circle $C_2 : |z| = 2$

   Let $f_2(z) = 2z^5$, $g_2(z) = -6z^2 + z + 1$.

   Similarly, by Rouché's Thm., we get

   $2z^5 - 6z^2 + z + 1$ has 5 zeros, counting multiplicities inside C_2.

   Therefore, the equation has $5 - 2 = 3$ roots, counting multiplicities in the annulus $1 \le |z| < 2$.